CHARLES PEIRCE’S “PHILOSOPHY OF NOTATION”

I have been asked by the editors of Southern Review to write on the general topic of “interdisciplinarity.” It is an assignment with which I feel a little uneasy. For although, in some important respects, my work since the mid-1970s has been interdisciplinary in nature, in the sense that it crosses divides between standard disciplines as defined by the curriculum of the “modern” university (literature and philosophy, for example), I am dubious as to the possibility of studying interdisciplinarity as such. Nevertheless, there are particular problems, or fields of problems, that emerge at the common boundary of two disciplines, and new disciplines are apt to arise at them. In cases such as “industrial archaeology” or “medical anthropology,” for example, the methods and the premises of each will be combined to override the boundary between them; in others such as the “philosophy of literature,” literature is simply appropriated as an object within the purview of philosophy and constituted by the problematics characteristic of a philosophical investigation,1 whatever the disciplinary study of literature might wish to say on the matter. “Interdisciplinarity,” in so far as it captures our attention and demands whole issues of journals to be devoted to it, is not usefully exemplified by cases on either of these models. We use the term precisely — and strategically — in those cases where the boundary continues to be asserted, where appropriation is resisted, where both newly emergent objects and the problematics associated with them are difficult to define. In these cases, “interdisciplinarity” tends to name the flouting of disciplinary manners, and the adoption of a language that is both highly specialised and unstable, unable to define or impose its conventions except by devices of exclusion. It is “interdisciplinary” because it is not a discipline, in the precise sense that it cannot be taught. To the extent that it is taught and becomes a “subject” subject to institutional protocols, to the extent in particular that it is examinable, it is or becomes a new discipline. This is the current situation of semiotics, which is characterised by the heterogeneity of its methods, its objects, and its assumptions. It is true that the problems thrown up by this situation are my current stock in trade.

In the sense I have just outlined, “interdisciplinarity” cannot be studied “as such.” It is not general, and to suppose it is to lose the only premise with which problems emerging at the intersection of two disciplines can be delineated. An interdisciplinary problem, or object, or domain, is one where neither one “idiom” (in Lyotard’s sense) nor the other provides the language for its positing or, a fortiori, its analysis.2 It is a point at which translation stalls. To study it, therefore, is to study its rhetoric, the way it derives from two familiar discursive logics and requires work of them that they baulk at. It is to study the necessity, and
the impossibility, of translation itself. I offer the present reading as a case study.

Claiming to pursue a project inherited from Kant, but grafting into it a set of categories deriving from medieval logic, Charles Peirce’s semeiotic (his spelling and appellation) is a hybrid from the outset. But it becomes even more hybrid when Peirce attempts to line it up with a contemporary project that is discursively distinct from both medieval logic and post-Cartesian metaphysics. This is the project of formal logic, itself an interdisciplinary project in the sense that it adapts the methods and principles of mathematics to the problems of the truth calculus, and, so doing, defines an entirely new domain of problems incommensurate with the “logic” that had been subordinate to philosophical enquiry hitherto. At this point, with — first — the development of the idea of an algebra, and — second — the development of mathematical methods using ever more refined notational systems, are born set theory, modal logic, the relative logic, the modern theory of probability, not to mention the logic of mathematics itself ... At this point, then, the syllogistic no longer defined the field called logic; the indeterminacy of natural language semantic categories were at last able to be eliminated. Logic, as Peirce was pleased to say, had had its “Copernican revolution.” But at the very moment when logic was distinguishing itself from the methods and principles of philosophy in favour of those of mathematics — that is, where logic was self-consciously divesting itself of its metaphysical grounding — Peirce’s own project became an attempt to bring them together. A philosophy that did not use the new logic could not claim to be “exact” or “scientific,” but likewise a logic that refused to acknowledge the questions of metaphysics was formalistic and void. Peirce’s work defines itself as this project of synthesis, and semeiotic is his instrument.

This is a case study, then, on a pointed, not a random example. For such is the nature of formal notations, that they of necessity raise the question of translation. And such is the nature of exemplification that it of necessity mobilises the postulate of the general, which it represents in its generality through the offices of the particular. This postulate, I accept — for such is the nature of the genre of the reading — but I mark it sous nature.

In 1885, Peirce published one of his famous papers in mathematical logic, entitled “On the Algebra of Logic.” Its subtitle was “A Contribution to the Philosophy of Notation” (Collected Papers 3: 359-403). He defines the aim of the paper as follows:

In this paper, I purpose to develop an algebra adequate to the treatment of all problems of deductive logic, showing as I proceed what kinds of signs have necessarily to be employed at each stage of the development. I shall thus attain three objects. The first is the extension of the power of logical algebra over the whole of its proper realm. The second is the illustration of principles which
underlie all algebraic notation. The third is the enumeration of
the essentially different kinds of necessary inference... (3:365)

It is the second of these objects that interests me. As far as I can tell,
this is the first time Peirce applies his theory of signs to the analysis of a
notation, and I rather suspect that this is the first time in the whole small
history that I am interested in. What is remarkable about it at the outset
is that a notation is a rule-governed system of arbitrary signs. To
investigate its semiotics is therefore very like what Saussure was to do
with natural language some thirty years later.5

However, I do not wish to fall into the facile gesture to which Peirce
scholars as a race are susceptible. That is, I do not wish to argue that
Peirce did it first, or that the real origin of semiotics is to be found in this
paper, or that Peirce’s theory of the sign is larger and more generous, or
more theoretically powerful than the propositions that emerged from the
Saussurean heritage, or that they should therefore count as the foundation
of semiotics. The fact is that Peirce’s work had very little influence on the
theory of signs until the second half of this century; for this reason, there
is little point in giving him the role of origin and author. More importantly,
he was a formal logician, and in that capacity, his view of “natural” — i.e.
discursive — language is simply that there is nothing interesting to be
discovered by investigating it, and that symbolic notations are useful
precisely because they provide a method for translating out of the
unclarities and undecidabilities of the discursive modes in which we
undertake our familiar practices. What is interesting, rather, is that
Saussure did make the move that Peirce refused, viz., to apply the model
of a formal notation back on to the site of discursive language, with all the
interesting problems that ensued and kept people like me in jobs.

So the point of my paper is not to rewrite history to give the
unacknowledged genius his due. In the first instance, my focus is much
more narrowly authorial than that: I am interested in the moves Peirce
makes to produce a theory of signs, how it came about that, in his
philosophical and logical practice, this rather unpredictable project
emerged. Beyond that, however, I have two further concerns. One is to
demonstrate the general thesis I have been developing over the past few
years, that is, that the determinants of any theoretical practice are
themselves generic (see my “Peirce’s Barrister”). In this respect I shall
make the following quite simple argument, viz., that the generic, or
rhetorical determinants of mathematical logic of the nineteenth century
enable the emergence of a site not available in philosophy as it was then
practised. However, this site could not be exploited, that is, used to
develop a “general semiotics” without the intervention into this site of
philosophy. Hence my title, and Peirce’s subtitle: the “philosophy of
notation” arises at a point of intersection between philosophy and the
methods of mathematical logic. My second purpose is to investigate that
intersection. It is complex and uncomfortable, rich in suggestions and
rife with difficulties. Not to put too fine a point on it, I think there are
ways in which “the philosophy of notation” yields insights about the
paradoxes that constitute what we now call “semiotics.”
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I do not intend to go over the range of questions associated with the topic of genre; it would be neither practical nor necessary. Instead, I shall use a convenient short cut, which I derive from the classical sources. A discursive genre is, among other things, a set of characteristic toposi, that is, headings that organise subject matter. If this is the case, then a genre is characterised in part by the toposi it excludes. A topical definition of genre such as this is useful for understanding intellectual disciplines as genres, since a topos is specified by what we now call "topics." In this respect, the term "representation" stands for a fairly standard "topic" of philosophy. I shall refer to some passages from Kant's Critique of Pure Reason, in which Kant spells out what he means by "representation" as part of his overall project, to give an account of the metaphysical conditions of knowledge. First (314), "representation" is a generic term covering perception, concept, intuition, notion and idea. Notice that within the framework of a philosophical psychology, these are all kinds of psychological event. Second (152-53), representations of any kind are referred to the cogito, to the "I think," that is, they are the events or products of a subject. This subject is a universal, "anthropological" subject, where "anthropology" is taken as the opposite of "ethnological," i.e. universal as against culturally specified. The problem raised, following this, is spelled out in the third passage: if "representations" are subjective, how do they claim, let alone guarantee, their relation with an object? This is the nub of the Kantian speculation. Now Kant's Critique is the authoritative text for Peirce, and a number of Peirce's major concerns, both narrowly, within his semiotic, and more broadly across the range of his philosophy, are inspired by Kant. Hence, for example, Kant's distinction between the thing-in-itself, its existence, and its status as object, is crucial to Peirce, particularly in his uptake of Kant's argument that a representation determines its object, but not the existence of that object, and that the concept of an object in general is a necessary entailment of representation (125). Similarly Kant argues that knowledge of objects is mediate, not immediate (105-06); it follows that representation must know itself as representation. I think Peirce's theory of the "representamen" (his technical term for the sign-vehicle) is significantly built up on these premises. Out of them, he will elaborate what he sometimes calls his "philosophy of representation," and at other times, "semiotic."

I also wish to refer to some passages from George Boole's An Investigation of the Laws of Thought. This is a major text in the development of mathematical logic, and it states the paradigm within which Peirce was to do his most important formal research. For my purposes, the most interesting thing in Boole's work is that following a first, introductory chapter on the "Design of the Work," its second chapter is devoted to "Signs and Their Laws." Notice first of all that "Language" is conceived to be an "instrument," "not merely a medium for the expression of thought" (24), but that the differences between particular "natural" languages are deemed to be irrelevant to the logician, who seeks to discover in their commonality and their universality "some deep
foundation of their agreement.” This deep foundation he takes to be “the
laws of the mind itself.” In order to investigate this foundation, Boole
will take it to be some universal form of language; he proposes “to give
expression in this treatise to the fundamental laws of reasoning in the
symbolic language of a Calculus” (5). If “Language” is the general
instrument of thought, this particular language is the special instrument
of the method of logic. It can be investigated so as to identify its elements,
and “to seek to determine their mutual relation and dependence” (24).
“The notation of the science of Number” (6) has “a peculiar and exclusive
fitness for the ends in view” (5), so to investigate the instrument of human
reason is to “inquire in what manner [its elements] contribute to the
attainment of the end to which, as co-ordinate parts of a system, they
have respect” (24). Boole’s view of the relation of method, instrument,
and the object of enquiry is the classic positivist one: “the laws [of
reasoning] are such as to suggest this mode of expression, and to give it”
its peculiar fitness to reveal the laws of the mind.

Now notice that the focus on instrument and method means that, for
Boole, certain metaphysical problems are in practice irrelevant. First, it
makes no difference whether “Language is to be regarded as an essential
instrument of reasoning, or whether, on the other hand, it is possible for
us to reason without its aid” (24). Whichever of these doctrines one
adopts, “the results obtained” from the actual investigation “are formally
equivalent” (25). This is because the logician is investigating “the laws
of signs,” and “the immediate subject of examination is Language, with
the rules which govern its use.” Ultimately, although Boole claims to be
investigating the laws of the mind, the ontology of mind, or its
transcendental condition, “is beside the design” of his work. Secondly,
he can afford to be agnostic about “a dispute as to the precise nature of
the representative office of words or symbols ... in the processes of
reasoning. By some it is maintained, that they represent the conceptions
of the mind alone; by others, that they represent things” (26). This is the
idealism vs. realism dispute; but again the logician declares that “The
question is of no great importance here, as its decision cannot affect the
laws according to which signs are employed” (26). With these two
declarations of irrelevance, Boole has set aside, as impertinent, the topos
of reason and the way it determines the investigation of knowledge and
truth, in a very long tradition of Western philosophy, and, most
particularly, in Kant. What is “impertinent” is a topoi, not on the agenda.
Boole has marked for us the boundary conditions for formal logic as a
specific discipline, and these boundaries are drawn so as to mark it off
from metaphysics.

My juxtaposition of Kant with Boole serves to show that the topic of
“representation” and the topic of “signs” are not the same. The topic of
representation is the topic whereby the constitution, and knowledge of
objects by a subjective instance, are investigated; entangled by this topic
are the problems of the ontology and the metaphysics of the mind and its
objects. Such questions as these do not need to be decided in order to do
the new logic; nor can this new logic contribute to any metaphysical
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... doctrine concerning them. Boole makes the distinction between the two topics quite clear in the very first sentence of his second chapter: the logician considers language as an instrument; as such, he investigates its elements, their systematicity, its fittedness to its end. This is, he says, wholly different from considering language merely as “a medium for the expression of thought” (24).

Another way of stating the difference between the topic of “representation” and the topic of “signs” is this: in so far as “representation” is involved in the metaphysics of the mind and its objects, the question of “representation” is the question of its content. What does a representation represent, and is it true? Implied is a “why?” question: to ask why we have, or produce, representations is to seek the nature of the human in its capacity for knowledge, and to ask why we might investigate representation is to seek the idea of philosophy itself. By contrast, the rhetoric of instruments and methods is the rhetoric of a technology: to ask “why?” of an instrument is to ask what pragmatic need it meets. Then again, to ask “how?” of representation is to seek the transcendental conditions of knowledge; this is Kant’s project. To ask “how?” of signs is to inquire into the techniques of reasoning and the rules of a system; this is Boole’s, and after him, Peirce’s.

I wish to call your attention to another point of interest in Boole’s project. He writes that “The elements of which all language consists are signs or symbols. Words are signs” (25). However, they are not the only sort of sign. “Arbitrary marks, which speak only to the eye, and arbitrary sounds or actions” are also signs, and “In the mathematical sciences, letters, and the symbols +, -, =, &c., are used as signs” (ibid.). He then invokes a distinction between “signs” and “symbols”: in the conventional usage of mathematics, “the term ‘sign’ is applied to the latter class of symbols, which represent operations or relations, rather than to the former [i.e. the ‘letters’], which represent the elements of number and quantity.” But he proposes to overlook this distinction, and uses “sign” as an overarching category, stipulating its usage by means of the definition. “A sign is an arbitrary mark, having a fixed interpretation, and susceptible of combination with other signs in subjection to fixed laws dependent upon their mutual interpretation” (25). In fact, the term “sign” is already a theoretical synthesis that disregards two conventional boundaries. One is the distinction between “numbers” and “operators,” and the other is the distinction between numerical notations and so-called natural, that is discursive language. These two oversights together construct the site for a general semiotics by constructing its theoretical object. This object classifies together, as “Language,” what we would standardly think of as two quite distinct instruments, “numbers” and “words”; it also classifies together, as “signs,” this first grouping with such things as the operators and “arbitrary marks... arbitrary sounds [and] actions.”

It is Peirce who will take up the suggestion of this extension of the category of “sign,” not Boole; but it is Boole who makes explicit in this passage exactly what an algebraic notation does by definition. It translates: discursive propositions are translated into a form which also translates
mathematical propositions. This leads to the comparison of the formal
laws of both systems, such that Boole claims to have shown that “the
ultimate laws of Logic are mathematical in their form,” and such that, by
the end of the century, that proposition was to be reversed. Frege and
Russel are the names attached to that reversal, and Peirce, too, saw it
coming: at the end of the introduction to this paper he writes: “I even
hope that what I have done may prove a first step towards the resolution
of one of the main problems of logic, that of producing a method for the
discovery of methods in mathematics” (3: 364). Already in 1867, he
argued “that there are certain general propositions from which the truths
of mathematics follow syllogistically” (3: 20).

This process of translation, on which the vocation of a whole new
discipline hangs, gives a particular significance to the term “sign” that is
not implied in the term “representation.” A notation is a system of signs
that translates two languages into one another. This task, or strategy, of
translation is already built into the decision to term the elements of each
system indiscriminately “signs,” and to consider simply as sub-classes,
or species of “sign,” the functional variety of elements of each language.
This means that the term “sign” and its nascent theory entail — as
emphatically the problematic of representation does not — the
wonderfully paradoxical problematic of translation. The paradox can be
stated simply, but it will be my business to elaborate on it somewhat in
later sections of my argument: translation is the construction of protocols
designed to produce, invent, or discover the form common to two distinct
languages, but the fact that we need to translate acknowledges at the
outset that those languages are not the same. Logic produces an instrument
and a method which simultaneously argues the sameness of the formal
structures of its two languages, and their difference. It follows that if the
“sign” names the act and fact of translation, the theory of signs will be the
theory of the differences as well as of the translatability of different
languages. We did not need to wait for Quine for this fact to be openly
acknowledged in the practices of mathematical logic and their
implications: we can find this acknowledgement in Boole. And it is no
mere concession. On it hangs the specificity of the whole enterprise of
formal logic. I quote the passage in full:

Now the actual investigations of the following pages exhibit
Logic, in its practical aspect, as a system of processes carried on
by the aid of symbols having a definite interpretation, and
subject to laws founded upon that interpretation alone. But at the
same time they exhibit those laws as identical in form with the
laws of the general symbols of algebra, with this single addition,
viz., that the symbols of Logic are further subject to a special law
(Chap. II), to which the symbols of quantity, as such, are not
subject. Upon the nature and the evidence of this law it is not
purposely here to dwell. These questions will be fully discussed
in a future page. But as constituting the essential ground of
difference between those forms of inference with which Logic is
conversant, and those which present themselves in the particular
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science of Number, the law in question is deserving of more than a passing notice. It may be said that it lies at the very foundation of general reasoning — that it governs those intellectual acts of conception or of imagination which are preliminary to the processes of logical deduction, and that it gives to the processes themselves much of their actual form and expression. It may hence be affirmed that this law constitutes the germ or seminal principle, of which every approximation to a general method in Logic is the more or less perfect development. (6-7).

There are several things about this passage that are of crucial importance for the later history of the discipline that Boole is launching. One thing that I suppose is taken for granted in that area is nonetheless worth stating in my domain of interest: Boole's discussion implies that formal similarity, as it is produced by the work of notational technologies, is the condition for the emergence of "difference": differences of interpretation, differences of language, differences of forms of inference, differences of sign from sign and of class of sign from class of sign. Ultimately, indeed, of "writing and difference."4 for as Boole explains, notations are written forms, they have to do with arbitrary marks upon a page; and Peirce remarks that they require new fonts, new technical inventions in the printeries, to do what they must do (cf. 3: 453).

Whereas Boole uses the term "sign" to subsume in the one category a variety of graphical objects which otherwise might be distinguished, Peirce construes — "translates" — this category as his category of representation. Within this concept he has elaborated a classification or "division" of the class into three: the symbol, the icon, and the index. While this classification is familiar to all readers of primers in semiotics, the highly technical use Peirce makes of it in the "Algebra of Logic" relies on a complexity unsuspected by theorists of the "iconicity of the image" or the "indexicality of the photograph." In my reading of Peirce's "philosophy of notation," I shall seek to show how the icon, that is, the sign that works by resemblance, takes on the task of producing the problematic of translation. I shall then go on to discuss the index, and I shall try to show that this sign, defined as working by "real connection," takes on the task of elaborating the written sign as the material conditions for the emergence of formal similarity and formal difference.

When Peirce proposes a "philosophy of notation" as one of the objectives of his "Algebra of Logic," he takes the interesting and unpredictable step of trying to synthesise the two topics I have distinguished by the words "representation" and "sign." He refuses Boole's declaration of the boundary between "philosophy and logic," and he will go on in later work to refuse the similar boundary between what is called "philosophical logic" and "formal (or mathematical) logic." In this latter enterprise, I suspect he was not successful; the former is the gamble of the paper I am interested in here. What I want to do is to scrutinise the text of the "philosophy of notation" to see how the two topics are brought together, and how they push against one another. This is a rhetorical investigation of the project of a general semiotics.
There are several tasks involved in this reading. One is the comparison of Peirce’s “rules” for deriving the three classes of sign in this text and in the first paper in which he proposed the classification. That paper is called “On a New List of Categories,” and was published in 1867 (1: 545-67). Its theoretical basis is a kind of speculative psychology, and the theory of signs derives from a phenomenological procedure whereby Peirce teases out the presuppositions of “comparison.” He shows that of the necessary elements of “comparison,” there are only three possible combinations, and these combinations give the three classes of sign. Something like this procedure is at work in “The Algebra of Logic,” but it has been formalised: “A sign is in a conjoint relation to the thing denoted and to the mind” (3: 360)—that is, it is a triple relation. The relata can combine in only three ways without violating the rules of the definition, and each of these ways is a kind or class of sign. Two features of this derivation interest me for my present purposes. One is that the general definition of “sign” accepts the metaphysical basis of the concept, and is indistinguishable from a definition of the topic of “representation.” Notice, then, that Peirce proposes a Kantian solution to the conundrum that Boole considers irrelevant to formal logic. Kant argues that it is not a question of choosing between idealism and objectivism (or any of the other dichotomous doctrines such as nominalism and realism), but that any viable philosophy of knowledge has to be an investigation of the conjoining. The second feature that interests me is that this theory of representation is then submitted to a sort of formal analysis. This fact carries two interesting implications. First, it is here that we can see the conjoining of the two topics, and the two rhetorics, that is, the refusal of the boundary that Boole draws; and secondly, because of this, we see Peirce’s determination to take seriously, as a philosophic problem, what Boole calls the “representative office” of any signs. Hence, the “philosophy of notation.”

I shall dwell at greater length on my second task. This is to read the process of exemplification in the paper. Peirce sets up his introduction to the topic of signs by first saying what he means by a “relation,” then showing why the concept of “sign” is properly analysed as a three-term relation. Then he derives the classes, and goes on to illustrate them. The important thing about the examples that he uses is that they fall neatly into two classes, those that are standard in his treatments of “representation,” and those that he introduces for the purposes of this paper. I have marked these latter with an asterisk. The chart shows examples from these texts, the first and third of which are philosophical, while the middle one is the “Algebra of Logic” that is the focus of my present attention.
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<table>
<thead>
<tr>
<th>1867 (1: 543-67)</th>
<th>1885</th>
<th>1903 (5: 7-165)</th>
</tr>
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<tbody>
<tr>
<td><strong>INDEX</strong> (murder)</td>
<td>natural signs</td>
<td>an old-fashioned</td>
</tr>
<tr>
<td>weathercock</td>
<td>physical symptoms</td>
<td>hygrometer</td>
</tr>
<tr>
<td></td>
<td>pointing finger</td>
<td>a land mark</td>
</tr>
<tr>
<td></td>
<td>*demonstrative</td>
<td>*a proper name</td>
</tr>
<tr>
<td></td>
<td>and relative pronouns</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*letters on a geometrical diagram</td>
<td>a pointing finger</td>
</tr>
<tr>
<td></td>
<td>*subscript numbers in algebra</td>
<td></td>
</tr>
<tr>
<td><strong>ICON</strong> (b/p)</td>
<td>*diagrams of geometry</td>
<td>statue of a centaur</td>
</tr>
<tr>
<td>portrait</td>
<td>painting</td>
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</tr>
<tr>
<td><strong>SYMBOL</strong> (man/homme)</td>
<td>general words</td>
<td></td>
</tr>
<tr>
<td>word, proposition</td>
<td>the main body of</td>
<td></td>
</tr>
<tr>
<td>barrister</td>
<td>speech, any mode</td>
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<td>of conveying a</td>
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<td>judgement</td>
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Notice that the examples for the class of the “symbol” remain constant through the three texts, and indeed, in all Peirce’s writings on “semiotic,” whether logical or philosophical. The “natural signs and physical symptoms” are quoted from a very old tradition in the topic of representation, which distinguishes between “natural” and “rational” signs, the “rational” signs being “language,” and the signs of the work of reason. There are some interesting things about the fact that this old distinction reappears in this text. First, an argument that is standardly accepted about Peirce’s philosophic enterprise is that it is a sustained attack on the dichotomy of mind and nature. Following Kant, what we know are the objects of our knowledge, not things in themselves, so the distinction of mind and nature becomes something of a nonsense; and following his own reading of Darwin, he is concerned to argue that in any case, the mind is a natural phenomenon and that scientific enquiry is an adaptive mechanism; the history of science, Peirce thinks, is an evolutionary phenomenon (see Wiener). Second, the reappearance of this distinction in this fairly primitive form is an indication of just how limited the philosophical part of the synthesis is forced to be in practice. It may well be that Peirce intends to investigate the “representative office” of algebraic signs, but he does not in this paper propose a “philosophy of representation.” However, I should add that this kind of parenthetical treatment of the big problems of philosophy is a rhetorical regularity of the writing of formal logic of the period.
There are some interesting adjustments to the standard examples in this paper. First, the "weathercock" disappears in favour of the "pointing finger." This may be a reduced version of the weathercock, since both are similar in that they "point" to indicate a direction in experiential space. But what is lost from the weathercock by focusing on this aspect is its status as a piece of technology, and it is this aspect that is retrieved in the text of 1903, with the example of the hygrometer. The weathercock and the hygrometer are technical instruments that construct data; neither is a "natural" sign, although it can be argued that they make "nature" available for inspection. The second important change is in the restriction of the class of the symbol. In the 1867 paper, Peirce does not discriminate between the functions of words: they are all words, and all "rational" signs; in the 1885 paper, however, this class is restricted to "the main body of speech," and it explicitly excludes "demonstrative and relative pronouns," which are now examples of the index. The "words" that continue to be symbols are "general words," that is, predicative words, and in 1903, Peirce adds to this the things we make with language, the discursive objects he calls "sentences and books." In the 1885 text, he generalises the proposition to include "any mode of conveying a judgement," that is, I suppose, mathematical propositions at least.

Note, now, that the criteria for each class — and indeed for the classification as a whole — are adjusted to allow inclusion of the new examples. The crucial fact is that all the new examples fall into the classes of the index and the icon. Now these two classes are excluded from the work of logic in Peirce's earlier, philosophical treatment of representation. In the "New List," Peirce writes:

The objects of the understanding, considered as representations, are symbols, that is, signs which are at least potentially general. But the rules of logic hold good of any symbols, of those which are written or spoken as well as those which are thought. They have no immediate application to likenesses or indices, because no arguments can be constructed of these alone, but do apply to all symbols. (1: 559).

I shall use this fact as another boundary marker between genres. At least in Peirce's practice, it is absolutely regular that what we might call "philosophical logic" — the philosophy of representation — limits its enquiry to the field of symbols. He develops his investigation of icons and indices only in his work on notational systems. While he does make some interesting attempts to carry over some of the results of these investigations into his philosophy, the restriction remains very much as I have suggested throughout his writing. What we find is that the work on these classes goes a great deal further in this paper than could be predicted by the standard distinctions between "natural" and "rational" signs that we find in traditional treatments of representation, or between "motivated" and "arbitrary" signs that we find in the post-Saussurean tradition.

Consider the criteria for the icon. The first is the standard one that subsists in almost all Peirce's expositions of the classification: the relation
of sign to object "consists in a mere resemblance between them" (3: 362). Then he elaborates: "icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry" (ibid.). This may well be the case for a diagram, this business of substitution; but is it so for a portrait? There may be cases — for example, portraits of monarchs on coins, or in ceremonial settings such as state funerals (drawn to my attention by Barbara Garlick) — which genuinely do "substitute" for their originals, but this is a matter of genre. It may also be a revealing thing to say about maps and architectural plans, that in the way we use them — to make calculations, plan a journey, etc. — we substitute the icon for its object. The more I think about this criterion of "substitution," the more I am convinced the Peirce was on to something fairly clever about the use of signs. This is rather a different matter from assessing how precise a resemblance is; evaluation of the degree, or quality, of iconicity requires us not to substitute the copy for the object. So let us suppose that Peirce is pointing us towards the use angle; but if this is so, it is precisely because we do distinguish the icon from the object that we use it in certain ways. So "substituting" and "not distinguishing one from another" cannot hold together. At least not for maps or portraits. But for the diagrams of geometry, whose objects are ideal, the two criteria are not incompatible. This is what Peirce goes on to elucidate: in order to use the diagram, we have to forget that it is not the very thing. Then he tries to extend that argument to the painting: "So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream — not any particular existence, and yet not general. At that moment we are contemplating an icon" (3: 362). There is something odd about this: whereas Peirce wrote that in the case of the diagram, the diagram is for us "the very thing." In the case of the painting, we seem to treat the painting itself as transparent, we do not operate on the painting, we dream, he says, its object. In the example of the painting, "icon" names a pure mental content that results from forgetting the difference between reality and representation; in the example of the diagram, "icon" names a representational technique that makes appear an abstraction that cannot appear without it. Indeed, in order to use icons such as geometrical diagrams, the material reality of the diagram must take on thinghood in order to permit the observation and manipulation of the relations it displays. In marked contrast with this, Peirce also uses the term "icon" to say what he means by the idea evoked by a predicate word. So the question arises whether the icon is a dematerialised mental content, or a material form governed by representational protocols. This question challenges the ambition to a synthetic theory of this class of sign. Peirce’s answer differs, depending on the generic environment in which the "icon" is deployed.

Indeed, this is how I take the equivocation in this paragraph: when Peirce is considering the signs of mathematics, the icon does a job for him which is close to the inverse of the job it does in the philosophy of
representation. This is because the topic of representation is tied by its 
traditions, by its rhetorical practices, to the problem of mental events. If 
Peirce is to resolve this dilemma, he must acknowledge, first of all, that 
it is a dilemma, and that the icon is a privileged site for it. I suggest that 
what he does is to treat the icon as a two-way street between ideal entities 
such as geometrical figures or fictional entities such as centaurs, and real 
material objects such as diagrams and statues. To get to this point, he will 
have to acknowledge the thinghood of representations, that is, the 
formal-material properties of different "languages" and their consequent 
fittedness for particular tasks. Analogies between languages must then 
be constructed in order to produce instruments of translation. Iconicity 
will become the theory of techniques of resemblance. If it mediates 
difference, it does so through its technical artifact, not through any 
putative "nature" of its own or its relation with an object.

These implications are already available when Peirce adds algebraic 
icons to the geometrical ones. Consider the following passage:

For instance, take the syllogistic formula,

\[
\begin{align*}
\text{All } M & \text{ is } P \\
\text{S is } M & \\
\text{hence } S & \text{ is } P
\end{align*}
\]

This is really a diagram of the relations of S, M, and P. The fact 
that the middle term occurs in the two premises is actually 
exhibited, and this must be done or the notation will be of no 
value. As for algebra, the very idea of the art is that it presents 
formulae which can be manipulated, and that by observing the 
effects of such manipulation we find properties not to be otherwise 
discerned. In such discoveries we are guided by previous 
discoveries which are embodied in general formulae. These are 
patterns which we have the right to imitate in our procedure, and 
are the icons par excellence of algebra. The letters of applied algebra 
are usually tokens, but the \(x, y, z\), etc., of a general formula, such as

\[(x + y)z = xz + yz\]

are blanks to be filled in with tokens; they are indices of tokens. 
Such a formula might, it is true, be replaced by an abstractly stated rule 
(say that multiplication is distributive); but no application could be 
made of such an abstract statement without translating it into a sensible 
image. (3: 363, emphases added)

Icons have the power to exhibit relations where other kinds of sign do 
not; they do so because they are "sensible," i.e. have a material form on 
the page. Some properties cannot be otherwise displayed, and no 
anapplication can be made of the abstractly stated rule that uses discursive 
language rather than the formula. Now because he is a logician, not a 
linguist, Peirce thinks of discursive language as lacking material properties, 
but it is quite clear from this example that the discursive form actually 
obeys what becomes discernible in the "sensible image." So let us 
suppose, contra the habits of logic, but in conformity with the habits of 
contemporary semiotics, that what is going on here is a translation
between two differently constituted material forms, not, as Peirce puts it, between the abstract and the concrete. Then it is clear that what he calls iconicity, the two way street where we might pass in either direction between the sign and its object, is exactly the principle of translation. The icon displays what is common between two signs of different languages. It is this common structure that is not distinguished from the icon, and it is the icon that makes it appear. If this is the case, similarity, analogy, resemblance itself are not given by nature but made under strict protocols. Peirce mentions these protocols in expressions such as “we have the right to imitate” and “we are guided.”

These protocols are rules, which in other places in this paper Peirce calls “prescriptions of use” (3: 385), as well as “rights” (3: 391); they are the regularities of a technical practice, the exigencies of a purpose, and they are conditions of possibility for certain operations and their outcomes. Hence, this understanding of the icon, and of the fact that it is governed by rules, solves a puzzle that he expresses quite early on. How can it be, he asks, that mathematics is both strictly deductive, and is able to produce “as rich and apparently unending a series of surprising discoveries as any observational science” (3: 363)? The answer is to be found in the rules governing the construction and manipulation of icons, whether geometrical or algebraic: each “new icon... [brings] out new propositions” (3: 381).

What I find interesting about Peirce’s discussion of icons in this paper is that had he left it at the example of the painting, with its capacity to evoke a “dream,” a pure mental content, none of these implications would have emerged. This is partly because he knows nothing about painting, and a great deal about mathematics and formal logic. But it is also because the investigation of the properties of a notation has room for the explicit assumption that diagrams have both formal and material properties. Boole, we remember, is concerned to specify notational signs as “written,” and it is the arrangement on the page of the letters, not the properties of the letters themselves, that does the work. Peirce’s decision to do his semiotics on the question of notation allows him to apply the assumptions of iconicity to it, where Boole does not. It is this that allows him to focus on, instead of disregarding, the materiality of the sign, and to show that its formal properties are dependent on this materiality.

In the long haul, what is even more fascinating — and this is why I think the genre thesis has something to say about the writing of theory — is this: when he writes his philosophy, he continues to disregard the materiality of signs, the formal material differences of different languages, and to go on talking about pure mental contents. For this reason, the mere decision to apply the concept of iconicity to a problem in a notation does not explain how Peirce comes to acknowledge, or theorise, the formal-material thinghood of signs in general, or of the icon in particular.

To find an answer to this question we need to turn to the criteria for indexicality. The familiar criterion of “real connection” also widens its meaning so as to apply to the new examples that are adduced. Certainly the letters on a geometrical diagram are really connected with the points
that they label, but it is not quite clear how this applies to algebraic subscripts. And "real connection" is only true of the demonstrative and relative pronouns in a restricted set of cases of oral discourse. Notice, then, that "real" covers causality, as in the case of physical symptoms, spatial contiguity, as in geometry, and spatio-temporal connection as in the case of the demonstratives. Now if we consider written, as distinct from oral discourse, spatial contiguity is hardly the question, and the guarantee of connection is made not by anything like space or time, but by the rules of connected discourse and some rather more formal rules of grammar such as the anaphoric and cataphoric determination of person and number and gender. If this counts as "real connection" in Peirce's account of indexicality, then "real" has been dissociated from the "natural." There is a "reality" of the domain of operation of a language just as compelling as is physical space. To learn a language is to learn the rules that govern that sense of a "conventional real."

Now this is a consequence of extending the class of the index to particular signs in a notation, all of which, it is clear, are governed by precise rules and conventions. There is indeed a reality established by the conventions of such languages, and again, it is the reality of the material and formal medium on which a notation depends. The rules governing indexicals in, say, discursive language, are rules that make the particular occurrence of some indexical sign establish a referring relation with another particular occurrence of some sign. This class, then, also implies the thinghood — or the eventhood — of a sign. A long-term consequence of this will be that Peirce distinguishes between the rules of the material form of the sign itself (the type/token distinction) and the rules of its relation with its object. As Derrida argues, this makes the supposed "singularity" of indexicals a highly problematic notion ("Signature événement contexte"). In Peirce's work, this will mean that the traditional distinction between the "particular" and the "general," used to distinguish between the index and the symbol, ceases to do any analytic work.

So where and when an index occurs is what makes it indexical, but these things are governed by rules, and rules that pertain to the materiality of the medium of the language. Notice, then, that both the icon and the index are rule-governed, once Peirce applies his classification to a formal notation. The crucial consequence of this is that it is now impossible to specify the symbol by its conventionality. All the signs of the system are conventional; they are simply distinguished by different conventions governing the criteria of their functionality.

Peirce adds further criteria to his specification of the index, and they are of the greatest interest. The first of these is that an index "denotes without describing." What this means is that an index has the capacity of establishing a referring relation, of installing something as object, without predicating anything of that object, save that it is there. It is a device for conferring objecthood on occasion; nothing is known of its objects save that they are objects, posited by virtue of the referring relation itself. This seems to be an uptake of Kant, in that it distinguishes
the problem of the ontology of objects from the concept, or operation, of objecthood. But instead of relying on the mental representation of objecthood, Peirce says that objecthood is a function of certain rules governing certain signs in particular languages.

The second criterion Peirce adds seems to be an implication of the first. In order to accommodate algebraic subscripts, he writes that they "distinguish one value from another without saying what those values are" (3: 361). This is apparently analogous to "denoting without describing," but if so, it construes "denote" in an altogether unexpected way. For whereas "denote" would ordinarily denote the relation of a sign with its object, its construal by "distinguish" reorients it to mean the relation of not-ness between two objects. Something is not something else. So, since the thinghood of sign and object are mutually entailed in the definition of the index, this construal states the condition of possibility of something being a sign, of its being an indexical sign, or of its being the object of such a sign. Likewise, since the classification of signs is a "distinction of icons, indices and tokens," the same principle accounts for the possibility of being any sort of sign, and of being a sign as distinct, say, from a particle (3: 359). Further, the condition of possibility of repetition, that is, of the difference of a sign from itself on two occurrences, must also be the same, since, as Peirce writes elsewhere, "it is not in the least necessary that the spots should be of different kinds, so long as each is distinguishable from the others" (3: 423). It seems to follow, then, that difference is itself the index of signhood, and that the "index" is the theory of this principle.

I cannot stress too strongly the significance of this outcome; but I am left speechless at the need to explicate it. Allow me, then, simply to repeat it: "difference is itself the index of signhood, and the 'index' is the theory of this principle."

It could not have arrived at this outcome in his philosophy. "Representation" is the theory of what he calls "positive" knowledge; and it continues to carry with it the problematic from which it arises, the synthesis of the understanding, the generality of predicates, the verifiability of propositions referring to the world outside the mind.

I want to add another observation, and it will bring me back to my focus on theoretical synthesis, and hence, the projects and the rhetoric of interdisciplinarity. The theory of signs, I have said, is necessarily, at its point of origin and in the way notational systems define its vocation, the site for the problem of translation. But translation is not confined to the relations of two systems, such as numerals and discursive language. It also pertains to the relations of two technical, or scientific languages, where language means, among other things, their protocols for defining their objects, determining and solving their problems. So when Peirce proposes a synthesis of the theory of "representation" and the theory of "sign," the negotiation of the two theoretical languages—the rhetoric of philosophy and the rhetoric of mathematical logic—can itself be read as an exercise in translation.

About halfway through his paper, Peirce sums up his semiotic
findings. "The logical algebra thus far developed" contains eight kinds of signs (3: 385). He will go on from here to modify the algebra, replacing some of these signs, introducing new ones, and so on, in order to see what "new propositions" concerning the laws of logic he can discover. Now the "eight" kinds of sign do not correspond in any way with the three classes of sign defined by his philosophical introduction of this topic, and he goes through the exercise in this paragraph of distributing the eight kinds into the three classes. What I want to point out is that this process of reclassification of the "kinds" as discerned by logic into the "kinds" as discerned by philosophy is precisely what I mean by "translation." Further, it is effected by a process that conforms in every particular to what Peirce has developed in this paper as the theory of the icon. For the three classes of sign are here being used as a model — hence, an icon — which, when applied to the eight kinds, does exactly what icons are expected to do: it manipulates the material of the algebra so as to make "new propositions appear."

To consider this procedure as a translation is the opposite of considering it as a theoretical synthesis; but I want to suggest that these opposites are complementary, and that that complementarity is paradoxical. To consider what Peirce is doing in this paper as a translation is to adopt one half of the paradox of translation, viz., that "philosophy" and "logic" are not the same, that they are languages, even technologies or instruments, fitted each to its own purpose, and that these purposes are incommensurate. By contrast, to consider the process as a synthesis with a claim to theoretical generality is to adopt the other half of the paradox of translation: we would then focus on the commonality of the two investigations, and we would take this commonality as the problems, the theoretical objects of semiotics. We would even claim that these problems transcend the conditions under which they are stated, assuming, with a certain notion of translation, that there is meaning, a common ultimate structure, a ground of thought. As against this, I argue that the theory of signs, but not the theory of representation, shows why each half of the paradox necessarily entails the other. We can have no synthesis without translation; we can have no translation without the claim to be a translation. But no claim to be a translation can be made in the absence of the principle of indexicality and the formal material operations of particular indices; and no translation that claims to be a translation fails to propose a synthesis, grounded in the icons of its own technical procedures.

I hope to have shown that the "philosophy of notation" is necessarily a mixed genre, governed not only by the two topoi I have distinguished as "the sign" and "representation," but also by their incompatible consequences. "Representation" will drive in the direction of the universal conditions of cognition, and it is these that guarantee the consistency, without residue, of synthetic theories of meaning. "The sign," however, rests on the materiality of indices and icons, and provides the ground for the paradoxes of translation. My further task will be to enquire into the relation between this exemplary mixity and the observably undisciplined
Charles Peirce’s “Philosophy of Notation”

practices of semiotics as it is propounded in the journals, conferences and classrooms that proclaim its status as a discipline.

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1 I have in my sights here the problematics associated with analytic philosophy; the question is different in the dialectical tradition associated with “Continental” philosophy.

2 I shall take the problem of a disciplinary rhetoric and that of genre to be interchangeable for the purposes of this analysis. For some interesting work on the issue of disciplinary rhetorics, see Nelson et al.

3 Saussure is said to have derived his ideas for the description of natural languages from the work of the Polish logicians of this period. If this is so, there may exist in their work examples of the kind of project Peirce undertakes in this paper.

4 I allude of course to Jacques Derrida’s L’Écriture et la différence, but the point is closer to the argument of his De la grammatologie.

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