Multi-directional Vibration Analysis of Cricket bats

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**Abstract:** As a sport with long history and rich tradition, cricket has seen little enhancement with regards the performance of the cricket bat since its inception. Most of the recent work has focused on blade design variations, while radical improvement to the blade is limited by the rules of the game. The aim of this study is about mapping the best performance range of the bat and finding a way to improve the performance of the bat within the laws of the game. A Finite Element model of the cricket bat was developed. It was shown that the lower the dwell time (ball-bat contact time), the less the bat deflection and hence more energy is transferred to the ball. The simulated stiffening of the bat’s handle with fibre-reinforced rubber was shown to mimic this reduced dwell time phenomenon.

**INTRODUCTION**

The fact that no alteration to the material composition of the blade is permitted restricts cricket bat to embark to the latest sport enhancement technologies that have proven to benefit other games of sports such as baseball and tennis [1,2,3]. Only a few improved bat designs have proceeded to commercial production within the boundaries of the game rules.

Till now, the quality of a cricket bat is judged by subjective observation on the grain structure on the face of the blade rather than by scientific investigation [4,5] and even the commercially available improved designs were not accompanied by proper testing of the performance against agreed standards. Another study focuses on the Sweet Spot that is essentially an area of a sporting instrument that inflicts maximum velocity of the ball and minimum response to the holder of the instrument. This has been well discussed and indexed as the heart of bat dynamic performance [6,7,8,9,10]. Also, the Coefficient of Restitution is reported to increase as the second mode of bending oscillation frequency increases [11].

As the sweet spot is regarded as the heart of bat dynamic performance, it is interesting to observe that, as far as the authors have determined, no attempt at mapping the sweet spot region of a cricket bat has been reported. The research effort so far appears to be directed towards the blade. The handle options remain relatively unexplored and it is this component that is least constrained by the laws of the game. These gaps provide an opportunity for this study. Therefore, the aims of this study are:

(i) Mapping the best performance range of the bat, (ii) investigating the factors affecting bat dynamic performance.

The handle will be the initial focus since it is least restricted by the rules of the game. This focus will fill the gap left by other researchers such as Grant, Nixon and Knowles [12,13,14]. A combination of experimental analysis and finite element analysis has been
employed throughout this work.

THEORY

Using energy considerations, a blow of the cricket bat to the ball consists of two separate steps. The first one is ball-bat impact. This step begins when the ball touches the bat and finished when they separate. The second step is from when the ball leaves the bat till the cessation of the bats transient vibration.

The fact is, a certain amount of energy is absorbed by deformation of the bat and will be dissipated in bat vibration. Thus, the energy distribution is broadly defined in Equation (1):

\[ E_1 + E_b = E_2 + E_d + E_i + E_s + E_v \]  

(1)

Where \( E_1 \) - initial (pre-impact) kinetic energy of the ball, \( E_b \) - kinetic energy given by the blow of the bat, \( E_2 \) - final (post-impact) kinetic energy of the ball, \( E_d \) - energy stored in deformation of the ball after it leaves the bat, \( E_i \) - bat indentation energy, \( E_s \) - energy dissipated as a form of sound or heat during impact, \( E_v \) - energy absorbed by the deformation of the bat body (Includes, vibration energy & initial (flexure) overall bat deformation).

Thus, the performance of the bat on a certain blow, which is represented by e, can be found out by the following equation.

\[ e = E_2 / E_1 \]  

(2)

\( E_v \) is obtained from the work of Gugan [15]. The dwell time of the ball can be calculated by Equation (3)

\[ T = 2.9432 \left\{ 15 \frac{M_1 M_2}{M_1 + M_2} \left( X_1 + X_2 \right) \right\}^{0.4} U^{-0.2} \]  

(3)

Where,

\[ M_1, M_2 \] - mass of ball and bat respectively, \( R \) - radius of the ball, \( U \) - initial speed of the ball, \( X_1, X_2 \) - impact constant given by Equation (4)

\[ X_i = \left( 1 - \sigma_i^2 \right) / E_i \]  

(4)

Where \( \sigma \) : Poison ratio of the ball \( E \) - Elastic modulus of the ball \((i=1)\), \( \sigma \) : Poison ratio of the bat \( E \) : Elastic modulus of the bat \((i=2)\). Equation (3) shows that for certain ball and bat, the dwell time is proportional to the initial speed of the ball. For example, the dwell time of the ball with a speed of 6m/s is 1.2 ms when the bat is hit.

MATERIAL PROPERTY EXPERIMENTS

An impulse hammer was used [17] as an exciter and FFT (Fast Fourier Transform) analyzer was used to process the modal analysis. Seven points were impacted along the central longitudinal axis of the blade. Clamped handle boundary condition was applied in the test.

Material property experiments have been carried out in order to get accurate data for the Finite Element Modelling (FEM) modeling. Three different materials: willow of the blade, rubber and wood cane of the handle, have been tested for density, elastic modulus and
Poisson' ratio

Considering the fact that the cricket bat is heavily rolled during the manufacturing process, specimens have been taken from both surface region and central region and tested respectively. Table 1 shows the results of the tests.

Table 1 Mechanical properties for different components of the cricket bat.

<table>
<thead>
<tr>
<th>Material</th>
<th>Rubber</th>
<th>Wood Cane</th>
<th>Willow (surface)</th>
<th>Willow (center)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (Kg/m³)x1000</td>
<td>1.113</td>
<td>0.561</td>
<td>0.535</td>
<td>0.508</td>
</tr>
<tr>
<td>Elastic Modulus (G Pa)</td>
<td>0.006</td>
<td>4.34</td>
<td>6.67</td>
<td>6.14</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.42</td>
<td>0.32</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

FINITE ELEMENT MODELLING (FEM)

The COSMOS/2.5 FEM package, developed by Structural Research and Analysis Corporation (SRAC), was used to model the cricket bat because its ability to carry out time response calculations for the ball-bat impact action. The cricket bat model has the same dimension with the real bat used in the modal test and consists of three parts that represent the blade, rubble layer and wood cane respectively. The whole model used 19310 4-node elements with a mass of 1.129 Kg.

Two sets of boundary conditions were applied in the modelling as shown in Figure 1. The first set is same as the condition applied in experiment so that the results could be compared with each other. The second boundary condition sets zero translation and rotation to the nodes on the top end surface of the handle. This set is closer to the real condition because considering the very short time of the impact (1ms) and relatively large impact force (over 10000N) the batman cannot hold the bat as tight as what the clamps did in the test. All the analyses were carried out based on the second boundary condition.

MODEL VERIFICATION

The frequencies for the bat obtained from the modal analysis and two boundary conditions of computer model are listed in Table 2

Table 2 Frequencies of the first four modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experimental Modal Analysis(1)</th>
<th>FEM results (Bound. One) (2)</th>
<th>(1) &amp; (2)</th>
<th>FEM results (Bound.Two)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.00</td>
<td>16.52</td>
<td>+10.1%</td>
<td>13.44</td>
</tr>
<tr>
<td>2</td>
<td>222.85</td>
<td>208.10</td>
<td>-6.6%</td>
<td>125.67</td>
</tr>
<tr>
<td>3</td>
<td>672.22</td>
<td>605.81</td>
<td>-9.9%</td>
<td>474.26</td>
</tr>
<tr>
<td>4</td>
<td>1688.33</td>
<td>1338.42</td>
<td>-20.7%</td>
<td>842.45</td>
</tr>
</tbody>
</table>
Figure 1 Two boundary conditions.

It can be seen that the FEM model predicts the frequencies of the real bat quite well in the first three modes. It does not work very well for the higher mode. However, the excitation spectrum obtained from the modal analysis suggests that the first three modes supply over 95% of the total contribution to the vibration response of the bat. Therefore the FEM modeling is presented as a feasible analytical tool to predict the behavior of the cricket bat.

DEFLECTION

Figure 2 shows the deflection of the hit points when the ball leaves the bat surface. Data was acquired from 21 points along the central axial of the blade, which has intervals of 12mm, from 36mm from the top of blade (near the handle) to 516mm.

Figure 3 shows the first three modes shape of the bat. It can be seen in the charts that the deflection has a minimal value near the node of mode two. The figures show that flexible vibration Mode shapes affect the deflection curve. This point can be explained as follows. First, considering the dwell time of 1.2ms, the vibration of modes 1, 2 and 3 whose frequency is lower than 830Hz will be generated. Mode 1 cannot be omitted because it has no node. However, it is easy to understand from the shape of mode 1 that the closer the hit point to the handle the less vibration will be activated for mode 1. For mode 2, if the ball hits its node, the energy absorbed by this mode will be minimised. Thus, the minimal deflection occurs at the point near the node of mode 2 on the side of handle because the energy absorbed by mode 1 & 2 is minimised when this point is hit.
However, it should be noted that mode 3 does not affect the shape of the deflection curve recognizably. It is because the dwell time (1.2ms) is about twice as long as ¼ the period of mode 3 (0.527ms). In another words, from considering mode 3, the bat has recoiled before the ball leaves the bat and it gives part of the energy absorbed in the first one-fourth circle of vibration of this mode back to the ball. Therefore, the energy absorbed by mode 3 is relatively small so that even when its node is hit, it cannot reduce vibration on a large scale. This can be easily be proved by changing the ball’s dwell time. The result is shown in the Figure 4 with a dwell time of 0.6ms. The shape of the deflection curve is obviously affected by nodes of both modes 2 and 3.

From both Figures 2 & 4, it is seen that the maximum deflection is almost 4 times the minimal one. It means that the performance of bat varies considerably when different parts of the bat is hit. In another words, the location of the impact point determines the degree of vibration and performance of the bat.

**STRESS AND STRAIN ANALYSIS**

In a more comprehensive report by Zhu Bang Li [18] the stresses and strains were analysed as follows: The strains were observed to be significantly higher in the rubber component of the handle. Thus, the rubber layer, has a major contribution on energy absorbed during bat deformation. Thus, the handle should be regarded as a primary target for design innovation.

*Figure 4* Deflection at point of impact for a dwell time of 0.6ms
APPLICATION OF THE FEM MODEL

A design innovation of introducing fiber reinforced rubber layer is evaluated. The new rubber layer contains 5% of the Graphite (T-300) Fiber parallel to the long axial of the handle. The elastic modulus of the layer can be calculated by \[ E_x = V E_f + (1-V) E_m \] (5)

Where \( V \) is the fibre volume fraction. Subscripts, \( x \), \( f \) and \( m \) represent the composite, fibre and matrix respectively.

The mass density, Poisson’s ratio and damping coefficient of the layer can be considered as unchanged because only limited fibers were used. The results of the natural frequency and deflection curves with the ball velocity of 6m/s are given in Table 3 and Figure 4.

From [18], it is very clear that the introduction of fibre-reinforced rubber layer solves the uneven plot of stress and reduces strain of the handle noticeably by increasing the stiffness of the handle significantly. The data also shows that the frequencies of the first three modes increase considerably. It means under the same impact condition, the bat with new layer tends to undergo less deformation. Thus less energy tends to be absorbed by the bat. This is shown by the deflection curve in Figure 5, which has a broader low value of deflection compared to the other deflection curve that represents the bat with a normal rubber layered handle.
CONCLUSION AND FURTHER WORK

Separating the ball-bat impact into two steps helps to understand the nature of this phenomenon. The energy transformations in the first step determine what degree of vibration will be generated in the second step. At the same time, the amount of energy causing vibration in the second step has great affect on the coefficient of restitution (CoE) that decides the performance of a hit.

A method of calculating the bat-absorbed-energy by the deflection of the hitting point when the ball leaves the bat is created. Post-dynamic analysis of a FEM model shows the bat-absorbed-energy varies a lot by the location of the hit point. As identified in the discussion above, mode shape and nodes location have great affect on bat-absorbed-energy and this eventually determines the performance characteristics of the bat.

The purpose of using rubber layer is to decouple the handle from blade and increase the damping effect of bat. However, it deteriorates the stiffness of the handle and lowers the frequencies of the bat considerably. An innovation of fiber-reinforced rubber was evaluated and it shows its advantages by increasing the sweet region significantly while holding the property of high damping coefficient of the rubber.

The force applied on the bat during impact was assumed to have a triangular shape (on a force-time plot) and is only proportional to the initial speed. An analysis of force time relation by experiment will offer a better understanding of energy transformations during the ball-bat impact and this should enable a better method of performance evaluation of the cricket bat.
REFERENCES