Background
Transpower is the owner and operator of New Zealand’s electricity transmission grid, and is New Zealand’s system operator.

New Zealand is about to introduce an electricity ‘financial transmission rights’ (FTR) market. Transpower, through its ‘Energy Market Services’ or EMS group, is hoping to be appointed to the newly created FTR manager role.

A few hubs (or nodes) will be defined on the electrical grid, and FTRs between each hub pair will be auctioned.

The usual solution to this is to use a modified version of the complex grid dispatch model, on the full electrical grid.

An alternative is to determine the intra-hub capacities on the full grid model, but then auction FTR products directly against those capacities. Transpower has been exploring this approach and finds that it greatly simplifies the auction for smaller networks. Transpower has developed several algorithms and prototype models of this simplified network auction: the problem is how far we can ‘push’ the product-based representation to include multiple interdependent hub pairs, losses and multiple types of FTR products.

What makes the auction particularly interesting is that it includes ‘option’ FTRs where the owner has the right of exercising that option or not depending on its future financial value. The auction therefore has to allow for every possible combination in which awarded options could be exercised.

Correct prices are as critical as correct award amounts.

The two-node loss-less model
Consider the simplest case of a hub pair A-B. B is north of A, so we have two possible directions of energy price difference, northwards with price at B (PB) > price at A (PA), or southwards. We use the term energy price P, to distinguish it from auction price, being the prices at which auction awards clear.

We have two types of FTR products: obligations (OB), that pay proportional to the price difference in the product direction, and thus can go negative when the price gradient is reverse, and options (OP), that pay proportional to the price difference in the product direction when it is positive, but zero when it is negative.

<table>
<thead>
<tr>
<th></th>
<th>Northwards PB &gt; PA</th>
<th>Southwards PA &gt; PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBAB</td>
<td>PB – PA (positive)</td>
<td>PB – PA (negative)</td>
</tr>
<tr>
<td>OBBA</td>
<td>PA – PB (negative)</td>
<td>PA – PB (positive)</td>
</tr>
<tr>
<td>OPAB</td>
<td>PB – PA (positive)</td>
<td>zero</td>
</tr>
<tr>
<td>OPBA</td>
<td>zero</td>
<td>PA – PB (positive)</td>
</tr>
</tbody>
</table>

If PB = PA then all payments are zero

The hub pair AB has a maximum capacity (measured in megawatts), so products on AB have a quantity limit, which could be different northwards and southwards, QAB and QBA.

The constraints on our auction of the four products are then simply:

\[
\begin{align*}
\text{OPAB} + (\text{OBAB} - \text{OBBA}) & \leq QAB \\
\text{OPBA} + (\text{OBBA} - \text{OBAB}) & \leq QBA \\
\text{OBAB} & \geq 0, \text{OBBA} & \geq 0, \text{OPAB} & \geq 0, \text{OPBA} & \geq 0
\end{align*}
\]

Any number of auction participants can bid any number of price-quantity pairs or ‘tranches’ for each product. The objective function is to maximise the value of auction awards as bid.
We have four products with two critical constraints, with right hand sides QAB and QBA. However from this we can determine the auction clearing prices for each product through observing that:

» for obligations, that in one direction is in every respect the reverse of the other, so they are in effect equivalent to one product that could be +ve or –ve, hence the auction clearing price of one will be the negative of the other.

» for options, the auction clearing price of an option in one direction is the price of the option in the other direction plus the price of an obligation in the original direction.

The following diagram is an example of an auction outcome, showing the bid curves and the auction clearing quantities and prices:

![FTR Auction Bids and Awards](image)

Extending the lossless network

We can then simply extend the model to a lossless radial network A–B–C–D... where each adjacent hub pair is treated as a separate auction.

However we can also allow products between non-adjacent hub pairs, such as AC by extending the constraints corresponding to each hub pair and direction, for example:

\[ \text{OPAB} + (\text{OBAB} - \text{OBBA}) + \text{OPAC} + (\text{OBAC} - \text{OBCA}) \leq \text{QAB} \]

We can also extend the model into loop flows, such as a triangular network A–B–C–A. In such cases, as power flows drive the underlying relationships between the products, the inter-relationships between the products are no longer 1:1. That is, the coefficients of the product terms within each constraint will include non-unity coefficients, e.g.

\[ \text{OPAB} + (\text{OBAB} - \text{OBBA}) + \text{ABXAC} (\text{OPAC} + (\text{OBAC} - \text{OBCA})) \leq \text{QAB} \]

where \( \text{ABXAC} \) is a coefficient indicating the correlation at the margin between the AB and AC product sets. These coefficients for each pair of product sets can be readily calculated from the dispatch network.

In this way, the auction can be extended to inter-connected networks with loop flows. The number of constraints increases, to \( n \times n! \) where \( n \) is the number of hub pairs.

The network with losses

In reality, dispatch includes the dynamic calculation of network losses, so prices have a loss effect that is often significant, and the injection and offtake quantities are different for any hub pair in each direction. We refer to these as ‘unbalanced FTRs’, as opposed to the ‘balanced’ FTRs discussed above.

The above treatment of a lossless network is an approximation. Moreover, while it provides a simpler mechanism for auctioning obligation and option FTRs, the problem of auctioning obligation and option FTRs on a lossless network has been solved, so it is not of great practice use.

However, the problem of auctioning option FTRs on a network with losses has not been solved, so significant value could be added to FTR design in New Zealand and overseas if we could solve it. We have proved that we can make unbalanced FTRs ‘revenue adequate’ for the simple two-node model, but have not extended this to a radial system or, most importantly, to networks with losses.

Multiple types of FTR products

We have obligation and option FTRs. By carefully choosing the capacity auctioned, we maximise the likelihood that there will be sufficient funds (the market rentals) to honour the payments of each FTR, but we can never guarantee it. It is always possible that FTR payments will have to be scaled back.

In the future we may want to subdivide each of these into a number of products of different ‘firmness’, say gold, silver and bronze products, where any scaling will apply to the lower firmness products first. The higher firmness products, auctioned against a proportion of the total quantity available, would be expected to command a higher price at auction.

What we will bring

Transpower will be providing models of the above auctions in Excel (using the built in Solver) and in GAMS, plus the constraint sets used. (We have also written these models in AIMMS, which we use for our operational dispatch systems, but this cannot be used without expensive licenses).

We will also provide our proof for a two-hub network with losses. The two people who have developed the model so far, Conrad Edwards and Vladimir Krichtal, will be present.

The problem

MISG is asked to review our work and consider how it could be extended to a fuller network with loop flows and losses and to multiple types of FTR products.